Diploma Programme
Programme du diplôme
Programa del Diploma

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

1 May 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

15 pages
2224 – 7109

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

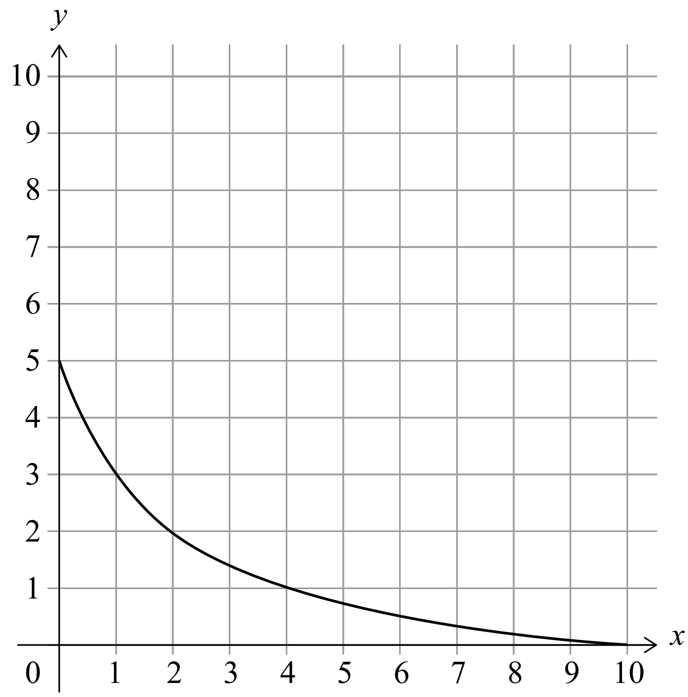
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The graph of $y = f(x)$ for $0 \le x \le 10$ is shown in the following diagram.

The graph intercepts the axes at $(10, 0)$ and $(0, 5)$.



(a) Write down the value of

(i) $f(4)$;

(ii) $f \circ f(4)$;

(iii) $f^{-1}(3)$. [3]

(b) On the axes above, sketch the graph of $y = f^{-1}(x)$. Show clearly where the graph intercepts the axes. [2]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark:4]

Solve $\tan(2x-5°)=1$ for $0° \le x \le 180°$.

3. [Maximum mark: 5]

(a) Solve $3m^2 + 5m - 2 = 0$. [3]

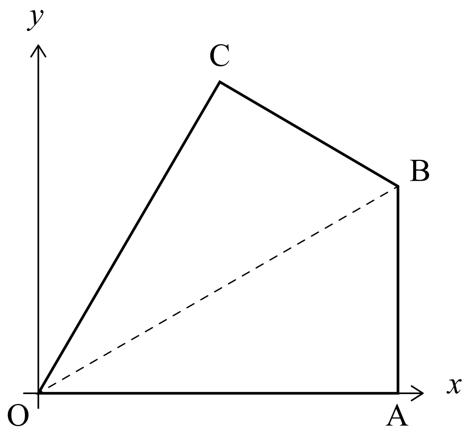
(b) Hence or otherwise, solve $3 \times 9^x + 5 \times 3^x - 2 = 0$. [2]

Please **do not** write on this page.

Answers written on this page
will not be marked.

4. [Maximum mark: 7]

Quadrilateral OABC is shown on the following set of axes.



OABC is symmetrical about [OB].

A has coordinates $(6, 0)$ and C has coordinates $\left(3, 3\sqrt{3}\right)$.

(a) (i) Write down the coordinates of the midpoint of [AC].

(ii) Hence or otherwise, find the equation of the line passing through the points O and B . [4]

(b) Given that [OA] is perpendicular to [AB], find the area of the quadrilateral OABC. [3]

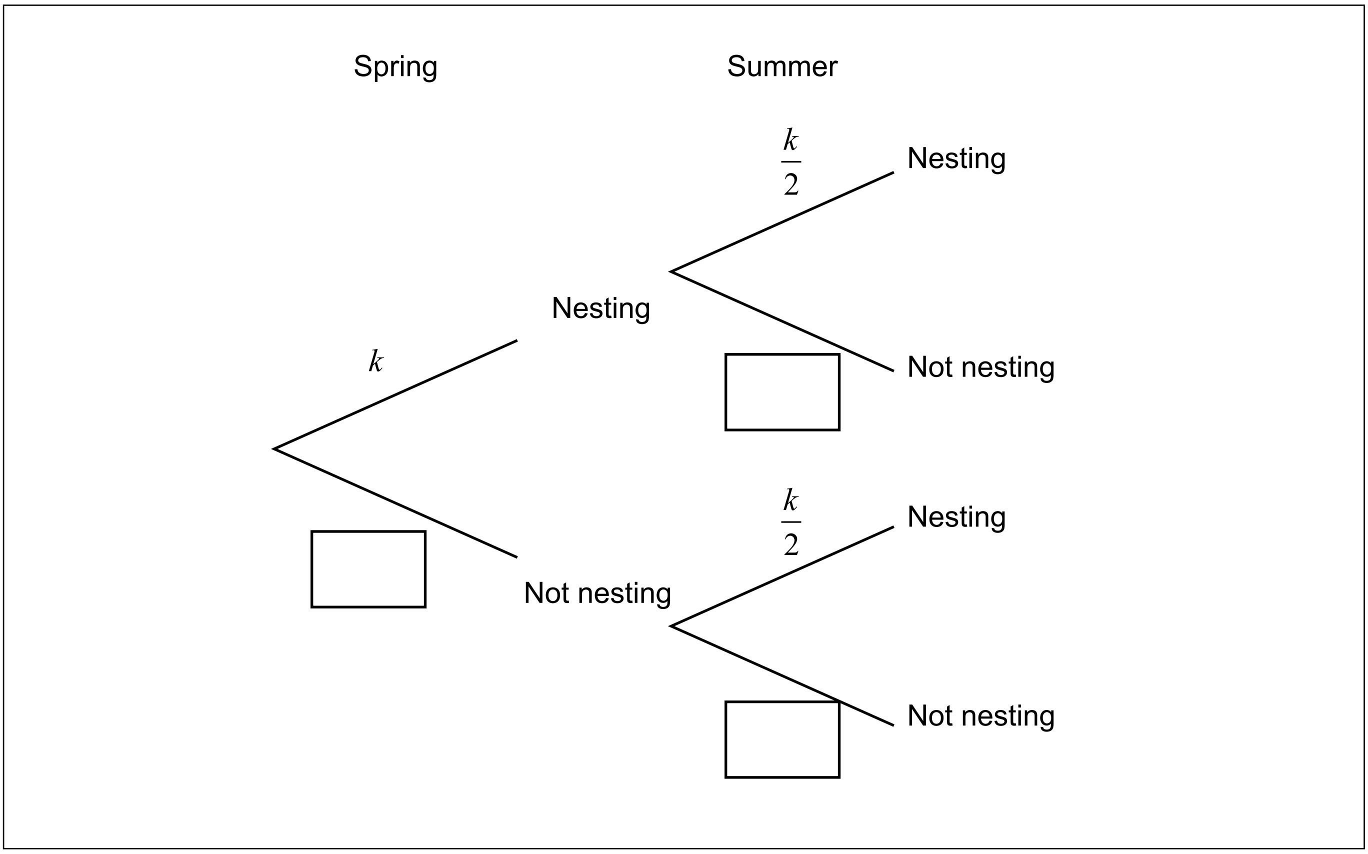
5. [Maximum mark: 6]

A species of bird can nest in two seasons: Spring and Summer.

The probability of nesting in Spring is k.

The probability of nesting in Summer is $\frac{k}{2}$.

This is shown in the following tree diagram.



(a) Complete the tree diagram to show the probabilities of not nesting in each season. Write your answers in terms of k. [2]

It is known that the probability of not nesting in Spring and not nesting in Summer is $\frac{5}{9}$.

(b) (i) Show that $9k^2 - 27k + 8 = 0$.

(ii) Both $k = \frac{1}{3}$ and $k = \frac{8}{3}$ satisfy $9k^2 - 27k + 8 = 0$.

State why $k = \frac{1}{3}$ is the only valid solution. [4]

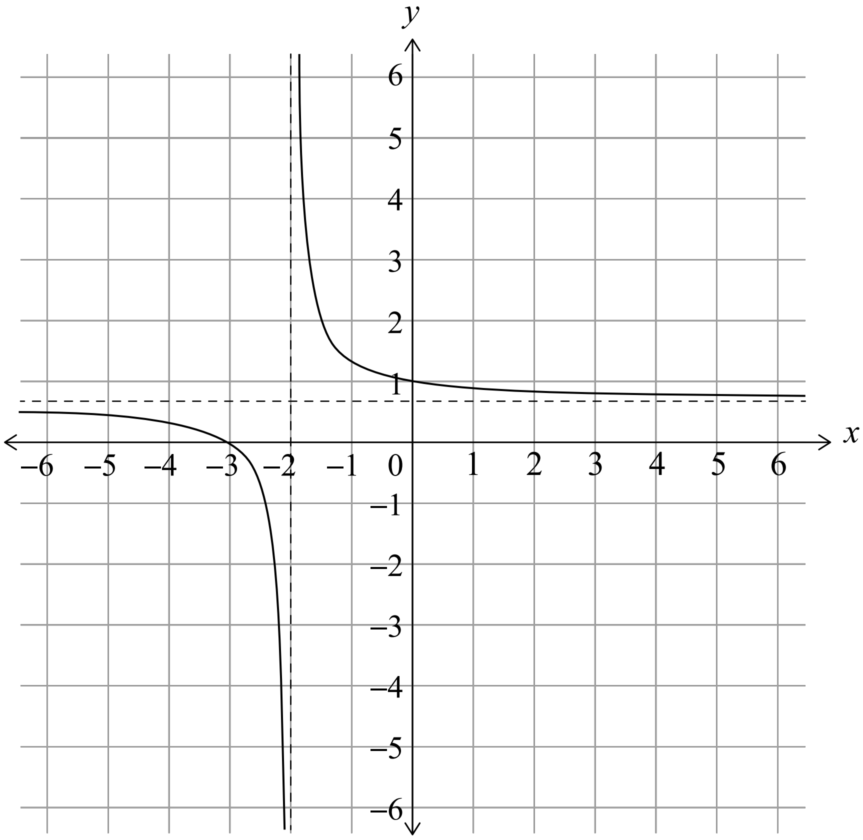
(This question continues on the following page)

(Question 5 continued)

6. [Maximum mark: 8]

A function f is defined by $f(x)=\dfrac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R}$, $x \neq -2$.

The graph $y=f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Consider $g(x)=mx+1$, where $m \in \mathbb{R}$, $m \neq 0$.

(b) (i) Write down the number of solutions to $f(x)=g(x)$ for $m>0$.

(ii) Determine the value of m such that $f(x)=g(x)$ has only one solution for x.

(iii) Determine the range of values for m, where $f(x)=g(x)$ has two solutions for $x \geq 0$. [7]

(This question continues on the following page)

(Question 6 continued)

Do **not** write solutions on this page.

Section B

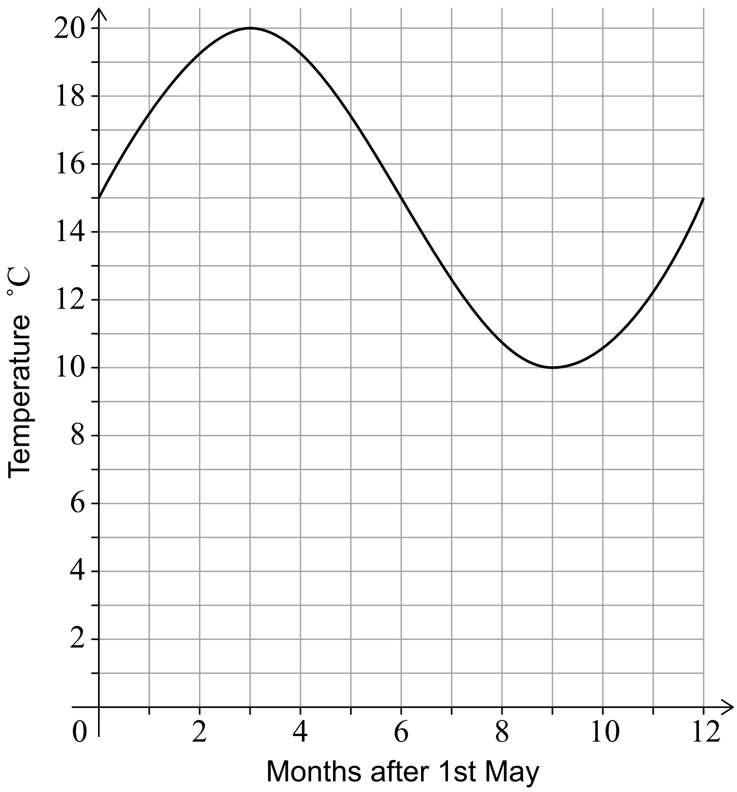
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 12]

Alex only swims in the sea if the water temperature is at least $15°\text{C}$. Alex goes into the sea close to home for the first time each year at the start of May when the water becomes warm enough.

Alex models the water temperature at midday with the function $f(x) = a\sin bx + \text{c}$ for $0 \le x \le 12$, where x is the number of months after 1st May and where a, b, $c > 0$.

The graph of $y = f(x)$ is shown in the following diagram.



(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Write down the value of

(i) a;

(ii) c. [2]

Alex is going on holiday and models the water temperature at midday in the sea at the holiday destination with the function $g(x) = 3.5\sin\frac{\pi}{6}x + 11$, where $0 \le x \le 12$ and x is the number of months after 1st May.

(c) Using this new model $g(x)$

(i) find the midday water temperature on 1st October, five months after 1st May.

(ii) show that the midday water temperature is never warm enough for Alex to swim. [6]

(d) Alex compares the two models and finds that $g(x) = 0.7f(x) + q$. Determine the value of q. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 17]

The derivative of a function f is given by $f'(x) = \dfrac{2x+2}{x^2+2x+2}$, for $x \in \mathbb{R}$.

(a) (i) Show that $x^2 + 2x + 2 > 0$ for all values of x.

(ii) Hence, find the values of x for which f is increasing. [3]

(b) (i) Write down the value of x for which $f'(x) = 0$.

(ii) Show that $f''(x) = \dfrac{-2x^2 - 4x}{(x^2+2x+2)^2}$.

(iii) Hence, justify that the value of x found in part (b)(i) corresponds to a local minimum point on the graph of f. [7]

It is given that $f(2) = 3 + \ln 10$.

(c) Find an expression for $f(x)$. [4]

(d) Find the equation of the normal to the graph of f at $(2, 3 + \ln 10)$. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

Consider the arithmetic sequence a, p, q ..., where a, p, $q \neq 0$.

(a) Show that $2p - q = a$. [2]

Consider the geometric sequence a, s, t ..., where a, s, $t \neq 0$.

(b) Show that $s^2 = at$. [2]

The first term of both sequences is a.

It is given that $q = t = 1$.

(c) Show that $p > \frac{1}{2}$. [2]

Consider the case where $a = 9$, $s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the

(i) arithmetic sequence;

(ii) geometric sequence. [4]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n.

The first three terms of u_n are $u_1 = 9 + \ln 9$, $u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

(e) (i) Find the common difference of the new sequence in terms of $\ln 3$.

(ii) Show that $\sum_{i=1}^{10} u_i = -90 - 25\ln 3$. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.